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Stable Massive States in 1+1 Dimensional NCOS

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Abstract

We study the case of open string theory on a D1-brane in a critical electric field. It was argued in hep-th/0006085 that the massless open string modes of this theory decouple from the massive modes, corresponding to a decoupling of the U(1) degrees of freedom in the U(N) gauge theory dual. To provide further support for the decoupling, we present several examples of open string disk amplitudes. Because of the decoupling, many of the massive open string states are stable, indicating the presence of a number of correspondingly stable states in the gauge theory. We provide a lengthy list of these stable states. However, when the theory is compactified in the spatial direction of the electric field, we demonstrate that sufficiently massive such states may decay into wound closed strings.

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1 Introduction

Non-commutative open string theory (NCOS) [2, 3] has been a subject of growing interest lately. This is the theory of open strings living on a D-brane in the presence of a critical electric field, which causes an infinite rescaling of the string tension and coupling. NCOS is in many ways very similar to the usual open string theory. Two important differences are, one, the amplitudes contain additional phase factors distinguishing different orderings of the vertex operators, and, two, the open string sector completely decouples from closed strings with zero winding number.

NCOS can live on a brane of arbitrary dimension, but we shall focus in this note on the case of the D-string. The electric field on a D-string is quantized: N units of electric flux describe a bound state of one D-string with N fundamental strings [4]. S-duality interchanges D-strings with fundamental strings, and hence, by S-duality, NCOS on a D-string is related to a U(N) Super Yang Mills (SYM) theory in a sector with one unit of electric flux [5, 6, 7, 1]. Furthermore, the gauge group U(N) factors into a product U(1)$\times$SU(N)$/Z_N$. The U(1) factor is free, and in this way duality predicts that there must be corresponding decoupling in the NCOS [1].

The authors of [1] showed that it is the massless open strings which decouple, and they presented a general argument involving the structure of the Green's functions. Here we show that, in the absence of a tachyon, a consequence of the decoupling of the massless modes is that a large class of the massive modes become stable. All decay channels involving massless scalars are forbidden, and for many massive modes, these decay channels are the only kinematically accessible ones.

However, when the theory is compactified along the spatial direction of the D-string, it is possible to create wound closed strings. These wound strings, unlike the strings with zero winding number, do not decouple from the open string sector. In fact, the wound strings provide an additional decay channel for the massive open strings. We show that the leading SO(8) Regge trajectory states (the states that carry maximum SO(8) spin for a given mass level), which would otherwise be stable, can decay into wound closed strings provided that energy conservation can be satisfied. As the mass of the lightest wound strings is proportional to the radius of compactification, we suspect that all of the stable states we find can decay into wound closed strings if the compactification radius is made small enough.

In support of their general Green's function argument for decoupling of the massless open strings, the authors of [1] present three disk amplitude calculations: the four-point amplitude for massless NS open superstrings, the forward scattering of a massless scalar off a tachyon in bosonic string theory, and the bosonic 4-tachyon amplitude. In this note, we tie up a loose end by considering the backward scattering of a tachyon off a massless scalar in bosonic string theory. We also present two superstring calculations involving a massive NS state. In particular we consider the $l = 1$ leading Regge trajectory state.

We calculate the two massive-two NS massless scalar amplitude and also the four-point massive amplitude. Because these open string amplitudes require less formalism and are easier to understand, we will present them before discussing amplitudes involving wound strings. But first, we begin with a discussion of the stable, massive open string modes.

2 Stable Massive Modes

In any scattering event, energy conservation must hold. Moreover, the mass level of an open superstring is given by $\alpha'_e m^2 = l$ where l is a non-negative integer. For a general n-body decay, energy conservation means

$$\sum_{i=1}^{n} \sqrt{l_i} \leq \sqrt{l} \tag{1}$$

where the letter i indexes the product states. In classifying the stable massive modes, we begin with an almost trivial observation. As the massless modes have decoupled, the only allowed decay products have an l of at least one, and the lowest energy decay state will consist of two such open string modes. Thus, by conservation of energy, all states with $l < 4$ are stable. When an $l = 4$ state decays, the two $l = 1$ modes are produced at threshold. In more than two dimensions, the absence of phase space for the final state would mean the $l = 4$ states are stable. However, in two dimensions, we can only say that, as the decay products have no energy to separate, we can never tell whether the $l = 4$ state has or has not decayed. With this caveat in mind, we will say that the $l = 4$ states are stable as well.

To proceed further in our classification of stable massive modes, we consider angular momentum. A more limited form of our argument was employed in [8]. We need some nontrivial results from representation theory, and for background, we refer the reader to [9]. In 9+1 dimensions, massive open string modes fall into irreps of SO(9). The classical irreps of SO(9) and also of SO(8) are described by Young tableaux with four rows. In what follows, we will ignore spinor representations. Following the notation of [9], we label Young tableaux with the Greek letters $\mu, \nu, \ldots$ and irreps by $\Gamma_\mu, \Gamma_\nu, \ldots$. The jth row of a Young tableau ν has ν_j boxes.

Consider an n-body decay of the state with Young tableau μ into states with Young tableaux $\nu \in P$ where all of the Young tableaux correspond to irreps of SO(9). In general, the relative motion of the decay products will transform under a representation of SO(9) which we will call R. Angular momentum conservation becomes the necessity that the irrep Γ_μ be included in the tensor product of the Γ_ν and of R:

$$\Gamma_\mu \in \left(\bigotimes_{\nu \in P} \Gamma_\nu \right) \otimes R.$$

However, R can be ignored. Our open strings are confined to a D-string. As a result, the relative motion of the decay products will not produce any angular momentum in the directions transverse to the D-string. This fact suggests that, given an open string mode with angular momentum in an irrep of SO(9), we should decompose it into irreps of SO(8) transverse to the D-string. R is trivial under this SO(8), and we can use the angular momentum condition

$$\Gamma_\mu \in \bigotimes_{\nu \in P} \Gamma_\nu . \tag{2}$$

where now Γ_μ and Γ_ν correspond to irreps of SO(8).

Let us demonstrate how this decomposition from SO(9) into SO(8) works in a simple case. The first excited NS sector state, ignoring ghost insertions, has a vertex operator in the minus one picture,

$$V = \zeta_{\mu\nu}\psi^\mu \partial X^\nu e^{ik\cdot X}$$

where ζ is in a traceless, symmetric representation of SO(9). Specifically, this state corresponds to a Young tableau with only one row and with two boxes and is an example of a state in the leading Regge trajectory. As representations of SO(8), the vertex operator decomposes into[1]

$$\begin{aligned} V_t &= \zeta_{ij}\psi^i \partial X^j e^{ik\cdot X} \\ V_v &= {\alpha'_e}^{1/2}\bar{k}_\alpha \xi_i (\psi^\alpha \partial X^i + \psi^i \partial X^\alpha) e^{ik\cdot X} \\ V_s &= (\alpha'_e \bar{k}\cdot\psi \bar{k}\cdot\partial X - \frac{1}{8}\delta_{ij}\psi^i \partial X^j) e^{ik\cdot X} \end{aligned}$$

where $\bar{k}^\alpha = \epsilon^{\alpha\beta}k_\beta$. The first NS massive state has decomposed into SO(8) irreps corresponding to Young tableaux with only one row and with two, one, and zero boxes. Indeed, this pattern will continue for the higher leading Regge trajectory states. In general, a leading Regge trajectory state at mass level l corresponds to a Young tableau ν such that $\nu_1 = l+1$ and $\nu_j = 0$ for $j > 1$. The leading Regge trajectory state at mass level l will decompose under SO(8) into Young tableaux with one row and with $l+1, l, \ldots, 0$ boxes. Each such tableau will appear only once in the decomposition, as can be verified by checking the dimensions.

For more general irreps of SO(9), there is a similar known decomposition. Let Γ_λ be an irrep of SO(9). Under SO(8), Γ_λ decomposes into $\bigoplus \Gamma_{\bar{\lambda}}$ where

$$\lambda_1 \geq \bar{\lambda}_1 \geq \lambda_2 \geq \bar{\lambda}_2 \geq \lambda_3 \geq \bar{\lambda}_3 \geq \lambda_4 \geq |\bar{\lambda}_4|.$$

The λ_i and $\bar{\lambda}_i$ are all integer or, for spinor representations, all half integer.

Therefore, to summarize what we should do to test if a particular decay is allowed by angular momentum conservation is to find out which irreps of SO(9) the original state

[1] Our indexing conventions are that $\alpha, \beta, \ldots \in (0,1)$ and $i, j, \ldots \in (2, 3, \ldots, 9)$.

and the decay products transform under. Next, we decompose these irreps into irreps of SO(8) according to the prescription given above. We note that the relative motion of the decay products cannot produce any SO(8) angular momentum, and so, we invoke condition (2). However, in what follows, we will not try to squeeze every last drop of information from this condition. Rather we will only consider what happens to the first row of the SO(8) Young tableaux under the tensor product. As a result, we will obtain a lengthy although probably incomplete list of stable states. For convenience, we relabel the length of the first row $\lambda_1 \equiv J$. The condition on the first row that we will prove now and use later is that

$$J \leq \sum_{i=1}^{n} J_i, \tag{3}$$

where the J without a subscript corresponds to the original open string state, and the i index the decay products.

In general, we expect to be able to decompose the tensor product of two irreps of any simple Lie group (and indeed of many other types of group as well) into a sum of other irreps

$$\Gamma_\lambda \otimes \Gamma_\mu = \bigoplus_\nu N_{\lambda\mu\nu}\Gamma_\nu.$$

R. C. King [10] has shown that, ignoring spinor representations, for both the symplectic and orthogonal groups, the multiplicities $N_{\lambda\mu\nu}$ are given by the formula

$$N_{\lambda\mu\nu} = \sum_{\zeta,\sigma,\tau} M_{\zeta\sigma\lambda} M_{\zeta\tau\mu} M_{\sigma\tau\nu}$$

where $M_{\lambda\mu\nu}$ is the corresponding multiplicity for the general linear group. A standard way of calculating M is to use the Littlewood-Richardson rules. Suppose that the Young tableau ν has J_ν boxes in the first row. It follows from these rules that for $M_{\lambda\mu\nu}$ to be nonzero the condition $\max(J_\lambda, J_\mu) \leq J_\nu \leq J_\lambda + J_\mu$ must hold. Then in our formula for N, for a term in the sum to be nonzero, $J_\sigma \leq J_\lambda$ and $J_\tau \leq J_\mu$, and also, $J_\sigma + J_\tau \geq J_\nu$. Putting these three inequalities together, we find that for $N_{\lambda\mu\nu}$ to be nonzero, $J_\nu \leq J_\lambda + J_\mu$. We have proven the desired result for the case of 2-body decay. The extension to n-body decay should be clear and follows by induction.

Condition (3) is most easily satisfied when the right hand side is as large as possible, i.e. when the first rows of the Young tableaux of the decay products are as long as possible. The leading SO(8) Regge trajectory states, by which we mean the first term in the SO(8) decomposition of the leading SO(9) Regge trajectory states, maximize this length at a given mass level l_i. Therefore, we set $J_i = l_i + 1$ and in its final form, angular momentum conservation for us will mean that

$$\sum_{i=1}^{n} l_i \geq J - n. \tag{4}$$

In what follows, we use $s = l + 1 - J$ instead of J.

First, let us consider the case $s = 0$. Squaring the conservation of energy inequality (1) and adding the condition from conservation of angular momentum (4), we find

$$\sum_{i<j} \sqrt{l_i l_j} \leq \frac{n-1}{2}.$$

As massless decay products are not allowed, $l_i \geq 1$. Thus, the left hand side of the above inequality is at least $n(n-1)/2$, and moreover $n \geq 2$. The inequality cannot be satisfied, indicating that all states with $s = 0$ are stable. In other words, the leading SO(8) Regge trajectory state is always stable.

Now let us consider the decay of a state $(l > 4, s)$ into n other open string modes. Note first that if our two inequalities (1) and (4) allow a $n > 2$ body decay, then an $n-1$ body decay is allowed as well. In particular, let two of the decay products be l_1 and l_2. We can always replace these two states with a third state l_3 such that $\sqrt{l_1} + \sqrt{l_2} \geq \sqrt{l_3}$ but also such that $(l_1 + 1) + (l_2 + 1) \leq l_3 + 1$. Therefore, we can restrict our analysis to two body decays.

For two body decay, the two inequalities are $\sqrt{l_1} + \sqrt{l_2} \leq \sqrt{l}$ and $s \geq l - l_1 - l_2 - 1$. To get a lower bound for s, we must maximize $l_1 + l_2$. We choose $l_1 \leq (\sqrt{l} - 1)^2$ and $l_2 = 1$, and find that, when $l > 4$, all states with

$$s < 2\sqrt{l} - 3 \tag{5}$$

are stable. In the case of equality, the state $(l, s = 2\sqrt{l} - 3)$ is stable because the decay products cannot separate. One interesting consequence is that for $s = 1$, all states are stable.

Here are two kinds of SO(9) irreps which, when decomposed, produce $s = 1$ states. The first irrep is a leading Regge trajectory state. When we decompose the leading Regge trajectory into SO(8), the second term in the sum, the term with $J = l$, has $s = 1$. The other kind of SO(9) irrep is the first subleading Regge trajectory, which is described by the Young tableau $(\nu_1 = l, \nu_2 = 1, \nu_3 = 1, \nu_4 = 0)$.

We can go on in this fashion. For example, for $s = 2$, only the states with $l = 5$ and 6 are possibly unstable. Note that in the case of the leading Regge trajectory, condition (5) is somewhat stronger because under SO(8), the leading Regge trajectory states decompose into Young tableaux with only one row.

3 Three Open String Disk Amplitudes

Before describing specific amplitudes, we take some time to describe how the electric field modifies what otherwise would be a completely straighforward and routine calculation in string perturbation theory. The modifications due to the electric field and the D-brane can be found in various places in the literature [11, 5, 12], but for reasons of clarity, we would like to present a brief summary here.

We turn on an electric field in the 01 directions with strength E. In the standard NCOS prescription, the string tension in the directions transverse to the D1-brane is set to $T_e = 1/(2\pi\alpha'_e)$ while $T = 1/(2\pi\alpha')$ is left as the tension in the 01 directions where $\alpha' = (1 - E^2)\alpha'_e$. One can show, for example from considering the Born-Infeld action, that the open string coupling constant is related to the usual closed string coupling g through $G_o^2 = g\sqrt{1 - E^2}$.

In the presence of an electric field, one would think that the open string vertex operators become modified. However, as long as we consider only open string scattering events, it is possible to work with the zero field vertex operators and absorb most of the effect of the electric field into the Green's functions. The one modification that we need to make to the vertex operators is to replace every α' with α'_e. The boundary Green's functions are then:

$$\langle X^\alpha(y) X^\beta(y') \rangle = -2\alpha'_e \eta^{\alpha\beta} \ln|y - y'| + i\frac{\theta}{2}\epsilon^{\alpha\beta}\mathrm{sgn}(y - y') \tag{6}$$

$$\langle \partial X^\mu(y) \partial X^\nu(y') \rangle = -\frac{\alpha'_e}{2}\frac{\eta^{\mu\nu}}{(y - y')^2} \tag{7}$$

$$\langle \psi^\mu(y) \psi^\nu(y') \rangle = \frac{\eta^{\mu\nu}}{y - y'} \tag{8}$$

where $\theta = 2\pi\alpha'_e E$. In the NCOS limit, $E \to 1$ and $\alpha' \to 0$ but α'_e and G_o are held fixed and finite [2, 3]. In the following sections we will confirm that it is the extra phases coming from (6) which lead to the decoupling of the massless modes in the NCOS limit.

3.1 Bosonic two tachyon-two massless scalar amplitude

The tachyon and massless scalar vertex operators are respectively

$$V_T = G_o e^{ik\cdot X} \quad \text{and} \quad V_S = \frac{G_o}{\sqrt{\alpha'_e}}\zeta \cdot \partial X e^{ip\cdot X}.$$

The mass shell conditions are $k^2 = 1/\alpha'_e$, $p^2 = 0$, and $p\cdot\zeta = 0$. Because we have restricted the ends of the open string to a D-string, ζ cannot have any component in the zero or one direction. This fact leads to simplification in the calculation of the amplitude — we need only contract the $\exp(ik \cdot X)$ and $\exp(ip \cdot X)$ operators among themselves.

The vanishing of the forward scattering of a scalar off a tachyon was calculated in [1], so we will look at backward scattering and choose the center of mass frame momenta to be

$$k_1 = \begin{pmatrix} e \\ p \end{pmatrix} \quad k_2 = \begin{pmatrix} p \\ -p \end{pmatrix} \quad k_3 = \begin{pmatrix} -e \\ p \end{pmatrix} \quad k_4 = \begin{pmatrix} -p \\ -p \end{pmatrix}$$

where $e^2 - p^2 = -1/\alpha'_e$. We choose

$$
\begin{aligned}
s &= -\alpha'_e(k_1 + k_2)^2 \\
t &= -\alpha'_e(k_1 + k_3)^2 \\
u &= -\alpha'_e(k_1 + k_4)^2.
\end{aligned}
$$

There are six possible orderings of the four vertex operators on the disk. The orderings pair up into the tu, su, and st channels. For a given pair, the phases created by the Green's function (6) are equal and opposite, resulting in a cosine function. Specifically, the phases are

$$
\begin{aligned}
tu: &\quad \cos\left[\pi\alpha'_e E(k_1 \wedge k_3 + k_2 \wedge k_4)\right] \\
su: &\quad \cos\left[\pi\alpha'_e E(k_1 \wedge k_2 + k_3 \wedge k_4)\right] \\
st: &\quad \cos\left[\pi\alpha'_e E(k_1 \wedge k_3 - k_2 \wedge k_4)\right].
\end{aligned}
$$

It is these phases which produce the new and interesting behavior in the NCOS limit. Summing over the three channels, the total amplitude is given by

$$
\begin{aligned}
&\frac{G_o^2}{\alpha'_e}\,\delta^2\left(\sum_i p_i\right)\zeta_1 \cdot \zeta_2\,(B(-u,-s)+ \\
&\qquad B(-1-t,-s)\cos[(s+1)\lambda] + B(-1-t,-u)\cos[(u+1)\lambda])
\end{aligned}
\tag{9}
$$

where $2\alpha'_e\lambda \equiv \theta$. The Beta functions are defined such that $B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$. In the critical field limit $\lambda = \pi$, it is not too difficult to see that the amplitude vanishes. The two crucial facts used in the calculation are $s+t+u = -2$ and the Gamma function identity

$$
\Gamma(x)\Gamma(1-x) = \frac{\pi}{\sin(\pi x)}.
$$

3.2 Massive Superstring Disk Amplitudes

As mentioned in the Introduction, we will consider the $l = 1$ leading Regge trajectory NS massive mode. The zero and minus one picture vertex operators for this mode are

$$
\begin{aligned}
V_{-1} &= \frac{G_o}{\sqrt{\alpha'_e}} e^{-\phi}\zeta_{ij}\psi^i\partial X^j e^{ik\cdot X}(y). \\
V_0 &= \frac{G_o}{\alpha'_e}\zeta_{ij}\left(\partial X^i\partial X^j - \alpha'_e\psi^i\partial\psi^j + i\alpha'_e(k\cdot\psi)\psi^i\partial X^j\right)e^{ik\cdot X}(y).
\end{aligned}
\tag{10}
$$

For simplicity, we are only considering the vertex operator which is a traceless symmetric tensor under SO(8). The mass shell condition is $k^2 = -1/\alpha'_e$.

We consider first the 4-massive amplitude. The center of mass frame momenta are

$$k_1 = \begin{pmatrix} e \\ p \end{pmatrix} \quad k_2 = \begin{pmatrix} e \\ -p \end{pmatrix} \quad k_3 = \begin{pmatrix} -e \\ -p \end{pmatrix} \quad k_4 = \begin{pmatrix} -e \\ p \end{pmatrix}.$$

There are a bewildering number of possible contractions to consider. In performing the calculation, we chose to put particles three and four in the zero picture. Also, making use of the $SL_2(R)$ symmetry, we sent $y_3 \to \infty$. Working through the combinatorics, the algebra, and the integrals, it turns out that only one of the contractions contributes to the amplitude. Specifically, it is a contraction involving the third term in each of the zero picture vertex operators. To cut a long story short, the amplitude simplifies to

$$\mathcal{A} \sim \frac{G_o^2}{\alpha'_e}\,\delta^2\left(\sum_i k_i\right)\pi(s-2)\frac{\cos(\pi s) - \cos(s\lambda\frac{p}{e})}{\sin(\pi s)}\mathrm{tr}(\zeta_1\cdot\zeta_3)\mathrm{tr}(\zeta_2\cdot\zeta_4). \tag{11}$$

The amplitude is similar to the bosonic 4-tachyon amplitude considered in [1]. As there are no massless particles involved, neither amplitude vanishes at critical electric field, $\theta = 2\pi\alpha'_e$. However, in this NCOS limit, both amplitudes do have a much softer UV behavior than the corresponding $\theta = 0$ amplitudes. Indeed, for large s, we may write:

$$\cos(\pi s) - \cos\left(\pi s\frac{p}{e}\right) = \cos(\pi(s-2)) - \cos\left(\pi\sqrt{s(s-4)}\right) \approx -2\pi\frac{\sin\left[\pi(s-2) - \pi(s-2)^{-1}\right]}{s-2}.$$

Therefore, for large s,

$$\mathcal{A} \sim -\frac{2\pi^2 G_o^2}{\alpha'_e}\,\delta^2\left(\sum_i k_i\right)\frac{\sin\left[\pi(s-2) - \pi(s-2)^{-1}\right]}{\sin[\pi(s-2)]}\mathrm{tr}(\zeta_1\cdot\zeta_3)\mathrm{tr}(\zeta_2\cdot\zeta_4)$$

which implies that the residues of the poles at $s = n$ fall off as $1/(n-2)$ while far from the poles, the amplitude is a constant. The softer behavior is not altogether surprising because we expect highly relativistic particles and massless particles to behave similarly. In [1] it was argued that the softened high-energy behavior is consistent with expectations based on perturbative gauge theory.

To conclude this subsection, and to provide additional support for the decoupling of the massless modes, we consider the forward and backward scattering of the SO(8) tensor NS massive mode off of a massless scalar. Because of the general argument presented in [1], the corresponding SO(8) vector and singlet massive modes should decouple from the massless scalar as well, but we will not check this fact. The NS sector massless scalars have zero and minus one picture vertex operators

$$\begin{aligned} V_{-1} &= G_o e^{-\phi}\xi\cdot\psi e^{ik\cdot X}(y) \\ V_0 &= \frac{G_o}{\sqrt{\alpha'_e}}\xi_\mu\left(\partial X^\mu + i\alpha'_e k\cdot\psi\psi^\mu\right)e^{ik\cdot X}(y). \end{aligned}$$

As for the bosonic massless scalar, $\xi \cdot k = 0$, $k^2 = 0$, and, as the open string is restricted to a D-string, ξ can have no components in the 01 directions. We chose to do our calculations with both massive vertex operators in the minus one picture.

We consider first the case of forward scattering:

$$k_1 = \begin{pmatrix} p \\ p \end{pmatrix} \quad k_2 = \begin{pmatrix} e \\ -p \end{pmatrix} \quad k_3 = \begin{pmatrix} -p \\ -p \end{pmatrix} \quad k_4 = \begin{pmatrix} -e \\ p \end{pmatrix}.$$

Because in forward scattering $t = 0$, only contractions between the bosonic pieces of the zero picture operators will contribute to the amplitude. The result is

$$A_f = \frac{G_o^2}{\alpha'_e} \delta^2 \left(\sum_i k_i \right) \pi(1-s) \frac{\cos(\pi s) + \cos[\lambda(1-s)]}{\sin(\pi s)} \xi_1 \cdot \xi_3 \mathrm{tr}(\zeta_2 \cdot \zeta_4). \tag{12}$$

This amplitude clearly vanishes in the limit $\theta = 2\pi\alpha'_e$ or equivalently $\lambda = \pi$.

Finally, we need to consider back scattering:

$$k_1 = \begin{pmatrix} p \\ p \end{pmatrix} \quad k_2 = \begin{pmatrix} e \\ -p \end{pmatrix} \quad k_3 = \begin{pmatrix} -p \\ p \end{pmatrix} \quad k_4 = \begin{pmatrix} -e \\ -p \end{pmatrix}.$$

The result is the most complicated in the paper:

$$\begin{aligned} A_b = \frac{G_o^2}{\alpha'_e} \delta^2 \left(\sum_i k_i \right) & ((1+t) B_1 \xi_1 \cdot \xi_3 \, \mathrm{tr}(\zeta_2 \cdot \zeta_4) + \\ & (B_2 - tB_3) \xi_1 \cdot \zeta_2 \cdot \zeta_4 \cdot \xi_3 + (B_4 - tB_5) \xi_1 \cdot \zeta_4 \cdot \zeta_2 \cdot \xi_3) \end{aligned} \tag{13}$$

where

$$\begin{aligned} B_1 &= B(2-u, 2-s) + B(-1-t, 2-s) \cos[\lambda(1-s)] + B(-1-t, 2-u) \cos[\lambda(1-u)] \\ B_2 &= B(2-u, -s) + B(1-t, -s) \cos[\lambda(1-s)] + B(1-t, 2-u) \cos[\lambda(1-u)] \\ B_3 &= B(2-u, 1-s) - B(-t, 1-s) \cos[\lambda(1-s)] + B(-t, 2-u) \cos[\lambda(1-u)] \\ B_4 &= B(-u, 2-s) + B(1-t, 2-s) \cos[\lambda(1-s)] + B(1-t, -u) \cos[\lambda(1-u)] \\ B_5 &= B(1-u, 2-s) + B(-t, 2-s) \cos[\lambda(1-s)] - B(-t, 1-u) \cos[\lambda(1-u)]. \end{aligned}$$

In the NCOS limit, the vanishing of all of these B_i is equivalent to the one Beta function identity

$$B(x, y) - B(z, x) \cos(\pi x) - B(z, y) \cos(\pi y) = 0$$

where $x + y + z = 1$ or equivalently here $s + t + u = 2$. This identity is the same one that appeared in demonstrating the vanishing of the back scattering of a bosonic tachyon off of a massless scalar.

4 Decay into Wound Closed Strings

In the uncompactified NCOS theory, the reason why the closed strings decouple from the open strings is largely kinematic: The open strings simply do not have enough energy to produce closed strings in the NCOS scaling limit, $\alpha' = \alpha'_e(1-E^2) \to 0$ and α'_e held fixed. However, when we compactify along the 1 direction with radius R, the wound closed strings with winding number w and n quanta of momentum in the compactified direction satisfy the onshell condition (see for example [13]):

$$-\alpha'(p_0)^2 - 2p_0 EwR + \frac{(wR)^2}{\alpha'_e} + \alpha'(n/R)^2 + 2(N_L+N_R) + \alpha'_e p_\perp^2 = 0,$$

where $p_\perp$ is the transverse momentum, along with a level matching condition $N_L - N_R = wn$. In the NCOS scaling limit, strings with nonzero w do not acquire infinite energy, and, as shown in [1], the dispersion relation becomes:

$$p_0 = \frac{wR}{2\alpha'_e} + \frac{\alpha'_e}{2wR}p_\perp^2 + \frac{N_L+N_R}{wR}.$$

If we consider the case of the wound graviton, $N_L = N_R = 0$ and $n = 0$, then, provided we make R small enough, $wR < 2\sqrt{l\alpha'_e}$, energy conservation will allow the decay of any massive open string state at level l into a wound graviton. Roughly speaking, this condition is equivalent to saying that a massive open string which is long enough to wrap around the compactified direction w times can turn, by joining its ends, into a wound closed string with winding number w.

In pure open string amplitudes, we were able to carry out the NCOS scaling limit at the level of Green's functions and vertex operators. Once we include wound states, we have to first work away from the scaling limit and take it only at the end. On the disk, we may use the doubling trick to express the $X(z,\bar{z})$ field in terms of its holomorphic part only [14]. In particular,

$$X^\mu(z,\bar{z}) = X^\mu(z) + (DM^{-1})^\mu{}_\nu X^\nu(\bar{z}).$$

where the M matrix comes from the boundary conditions imposed by the electric field on the open strings. In the directions transverse to the D-string, M is the identity, while parallel to the D-string

$$M^\alpha{}_\beta = \frac{1}{1-E^2}\begin{pmatrix} 1+E^2 & -2E \\ -2E & 1+E^2 \end{pmatrix}.$$

The D matrix is the identity in the directions parallel to the D-string and minus the identity in the directions transverse to the D-string. A lengthier discussion of these M and D matrices can be found in [15].

With this doubling trick in mind, the Green's functions we will use are the usual

$$\begin{aligned}
\langle X^\alpha(z)X^\beta(z')\rangle &= -\frac{\alpha'}{2}\eta^{\alpha\beta}\ln(z-z') \\
\langle X^i(z)X^j(z')\rangle &= -\frac{\alpha'_e}{2}\delta^{ij}\ln(z-z') \\
\langle \psi^\mu(z)\psi^\nu(z')\rangle &= \frac{\eta^{\mu\nu}}{z-z'}\ ,
\end{aligned}$$

where $\alpha, \beta = 0, 1$ and $i, j = 2, \ldots, 9$. As we are considering a two point function, the phases will not contribute.

Let us consider an open string state on the leading Regge trajectory decaying from rest. It has no momentum in the spatial direction: $k_1 = 0$. Although we have done the calculation for more general polarization tensors, for clarity we will present the results only for states polarized transversely to the D-string, i.e. the states on the leading SO(8) Regge trajectory. In section 2 we showed that all such states are stable in the uncompactified theory. Now we demonstrate that, when the spatial direction is compact, sufficiently heavy such states can decay into wound closed strings. It is enough to consider winding states of the graviton polarized transversely to the D-string. We take the open string vertex operator in the zero picture while the wound graviton will be in the $(-1,-1)$ picture to saturate the ghost number on the disk: [2]

$$\begin{aligned}
V_{m,0} &= G_o\left(\frac{2}{\alpha'_e}\right)^{\frac{l+1}{2}}\frac{1}{\sqrt{l!}}\xi_{ij_1\ldots j_l}\Big(\partial X^i\partial X^{j_1}\ldots\partial X^{j_l}+ \\
&\quad -\alpha'_e l\ \psi^i\partial\psi^{j_1}\partial X^{j_2}\ldots\partial X^{j_l} + \\
&\quad +i\alpha'_e(k\cdot\psi)\psi^i\partial X^{j_1}\ldots\partial X^{j_l}\Big)\, e^{ik\cdot(1+M^{-1})\cdot X}(y) \qquad (14) \\
V_{g,-1,-1} &= G_o^2\epsilon_{ij}\left(e^{-\phi}\psi^j e^{ip_R\cdot DM^{-1}\cdot X}\right)(\bar z)\left(e^{-\phi}\psi^i e^{ip_L\cdot X}\right)(z)\ . \qquad (15)
\end{aligned}$$

The polarization tensors $\xi_{ij_1\ldots j_l}$ and ϵ_{ij} are completely symmetric and traceless. Because we have compactified, we need to use the right and left momenta $p_{0L} = p_{0R} = p_0 + EwR/\alpha'$, $p_{1L,R} = \pm wR/\alpha'$ and $p_{iL} = p_{iR} = p_i$. On shell, $-k^2 = (k_0)^2 = l/\alpha'_e$, and energy conservation means that $k_0 = -p_0$.

In calculating the amplitude, only the second term in (14) will contribute to the contractions. The amplitude is coordinate independent, as we would expect from $SL_2(R)$ invariance:

$$\sqrt{l!}\ \mathcal{A}(k,\xi;p,\epsilon) \ \sim\ 2l\frac{G_o}{\alpha'_e}\left(\frac{\alpha'_e}{2}\right)^{(l-1)/2}\xi_{ij_1\ldots j_l}\epsilon^{ij_1}p^{j_2}\ldots p^{j_l} \qquad (16)$$

[2] This normalization of the winding vertex operator produces finite amplitudes on a sphere, with effective interaction strength of the winding states $\sim G_o^2$. We are grateful to M. Krasnitz for helpful discussions of this issue.

In the NCOS limit, the amplitude is finite. Therefore, it is indeed possible for sufficiently heavy leading Regge trajectory states to decay into wound gravitons. We conjecture that the same holds true more generally: Even if decay into lighter open string states is prohibited, any massive open string state can decay into lighter wound closed string states.

5 Remarks

The general argument presented in [1] along with the many open string scattering amplitudes calculated both in [1] and in this paper show that the massless open string modes do indeed decouple from the rest of the open string spectrum. As we saw here, one interesting consequence of the decoupling is that a number of the massive open string modes become stable in the uncompactified theory. It is interesting to speculate as to what these stable states correspond in the dual U(N) gauge theory. One possibility is a set of nontopological solitons: localized lumps of finite energy. Roughly speaking, such a lump is a section of an individual D-string which becomes liberated from the bound state and spins around it with sufficiently high SO(8) angular momentum. In the compactified theory, sufficiently highly excited such states may decay into Higgs branch excitations; there is enough energy for the entire D-string to become liberated. This process is dual to the transition from an open to a wound closed string in NCOS that was studied in section 4.

For large N the dual NCOS is weakly coupled. Hence the stable states are predicted to have $m^2 = n g_{YM}^2/N^2$, where n are integers. For finite N further corrections in $G_o^2 = 1/N$ should appear, and the spectrum will no longer be linear. It seems likely, however, that some of the states will remain stable for all $N > 1$. Perhaps these stable states may be found through numerical simulations of the gauge theory with an electric flux line.

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